"Particle number production from the interaction Hamiltonian with non-equilibrium quantum field theory"

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introduction

* Study mechanism of the particle number production with non-equilibrium quantum field theory in our model which is CP violating and breaks the conservation of particle number.

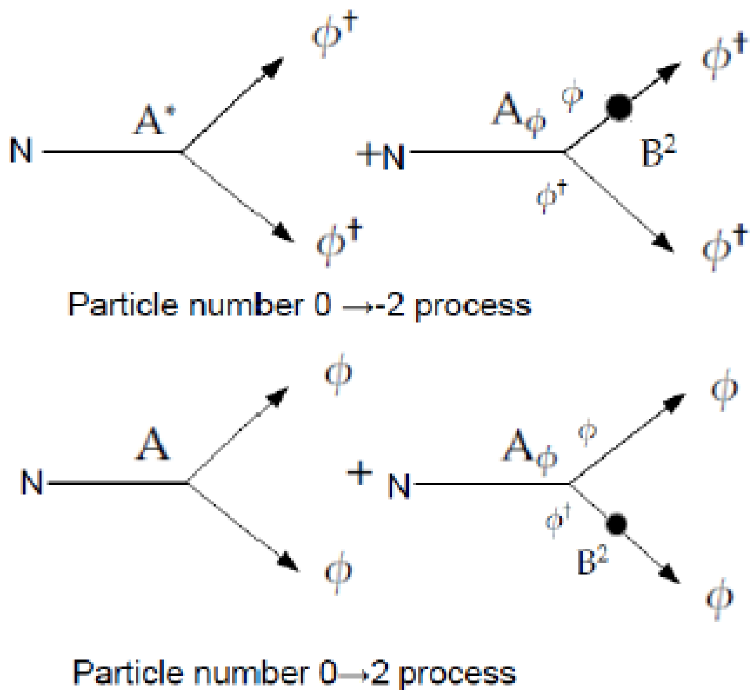
- In flat space and in thermal equilibrium, we know that particle number production doesn’t happen.

- We show time evolution of the contribution from the decay process.

Model

Lagrangian of our model

$$\mathcal{L} = \frac{1}{2}\partial_\mu N \partial^\mu N - \frac{1}{2}m_N^2 N^2 + |\partial_\mu \phi|^2 - m_\phi^2|\phi|^2 + B^2\phi^2 + B^{*2}\phi^{*2} + AN\phi^2 + A^* N\phi^{*2} + A_\phi N\phi\phi^*,$$



- The rate difference between top and bottom processes produces the net particle number.
- The heavy neutral scalar N has particle number 0.
- The light complex scalar Φ and Φ† have particle number -1 and 1.
- The relatively phase of A and B^2 is a CP violating phase.

Real field formalism

Rewrite the Lagrangian by using real scalar fields Φ1 and Φ2.

$$\phi = \frac{\phi_1 + i\phi_2}{2} \qquad \phi^\dagger = \frac{\phi_1 - i\phi_2}{2}$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu N \partial^\mu N - \frac{1}{2} m_N^2 N^2 + \frac{1}{2}\sum_{i=1}^{2} \partial_\mu \phi_i \partial^\mu \phi_i + \frac{1}{2}\sum_{i=1}^{2} m_i^2 \phi_i^2 + \phi_i A_{ij} \phi_j N$$

$$m_1 = \sqrt{m_\phi^2 - B^2} \quad m_2 = \sqrt{m_\phi^2 + B^2} \qquad A_{ij} = \begin{pmatrix} |A|\cos[\phi_A] + \frac{A_\phi}{2} & -|A|\sin[\phi_A] \\ -|A|\sin[\phi_A] & |A|\cos[\phi_A] - \frac{A_\phi}{2} \end{pmatrix}$$

Masses of real scalar are splitted into m1 and m2 by the influence interaction term of B.

Green function and Particle number

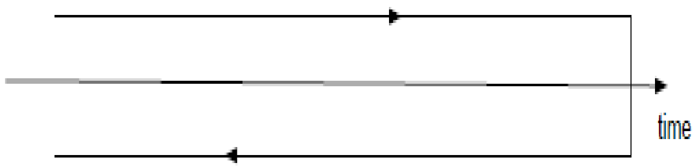
$$j^\mu(X) = i\Big(\phi^\dagger(X)\partial^\mu\phi(X) - \partial^\mu\phi^\dagger(X)\phi(X)\Big)$$
$$= \phi_2(X)\partial^\mu\phi_1(X) - \partial^\mu\phi_2(X)\phi_1(X)\Big)$$
$$= \Big(\partial^{x_\mu}G^{12}_{12}(x,y) - \partial^{y_\mu}G^{12}_{12}(x,y)\Big)|_{x=y=X}$$

Current divergence=production rate of particle number.

$$\frac{\partial}{\partial X^\mu}\langle j^\mu(X)\rangle = \Big(\partial_{x_\mu}\partial^{x_\mu} - \partial_{y_\mu}\partial^{y_\mu}\Big)G^{12}_{12}(x,y)|_{x=y=X}$$

$$\langle j^\mu(X)\rangle = Trj^\mu(X)\rho(0)$$

$$G^{11}_{ij}(x,y) = \langle T\phi^1_i(x)\phi^1_j(y)\rangle$$
$$G^{22}_{ij}(x,y) = \langle\tilde{T}\phi^2_i(x)\phi^2_j(y)\rangle$$
$$G^{12}_{ij}(x,y) = \langle\phi^2_j(y)\phi^1_i(x)\rangle$$
$$G^{21}_{ij}(x,y) = \langle\phi^2_i(x)\phi^1_j(y)\rangle$$



Density matrix in initial time

$$\rho(0) = \frac{\exp[-\beta(H_0 - \mu N)]}{Tr\,\exp[-\beta(H_0 - \mu N)]} = \frac{\exp[-\beta H_0]\exp[-\mu N]}{Tr\,\exp[-\beta H_0]\exp[-\mu N]}$$

Hamiltonian in Initial time commute particle number N. $N = \int d^3x j^0(X)$

$$\langle \phi^1 | \rho(0) | \phi^2 \rangle = \exp[\int d^4x \int d^4y K_{ij}(x,y)\phi_i(x)\phi_j(y)]$$

$$K(x,y) = -i\delta(x^0)\delta(y^0)\kappa(\mathbf{x}-\mathbf{y})$$

$$\kappa(\mathbf{k}) = \begin{pmatrix} -\frac{\cosh\beta\omega(\mathbf{k})}{\sinh\beta\omega(\mathbf{k})}\omega(\mathbf{k}) & \frac{\cosh\beta\mu}{\sinh\beta\omega(\mathbf{k})}\omega(\mathbf{k}) & 0 & i\frac{\sinh\beta\mu}{\sinh\beta\omega(\mathbf{k})}\omega(\mathbf{k}) \\ \frac{\cosh\beta\mu}{\sinh\beta\omega(\mathbf{k})}\omega(\mathbf{k}) & -\frac{\cosh\beta\omega(\mathbf{k})}{\sinh\beta\omega(\mathbf{k})}\omega(\mathbf{k}) & -i\frac{\sinh\beta\mu}{\sinh\beta\omega(\mathbf{k})}\omega(\mathbf{k}) & 0 \\ 0 & -i\frac{\sinh\beta\mu}{\sinh\beta\omega(\mathbf{k})}\omega(\mathbf{k}) & -\frac{\cosh\beta\omega(\mathbf{k})}{\sinh\beta\omega(\mathbf{k})}\omega(\mathbf{k}) & \frac{\cosh\beta\mu}{\sinh\beta\omega(\mathbf{k})}\omega(\mathbf{k}) \\ i\frac{\sinh\beta\mu}{\sinh\beta\omega(\mathbf{k})}\omega(\mathbf{k}) & 0 & \frac{\cosh\beta\mu}{\sinh\beta\omega(\mathbf{k})}\omega(\mathbf{k}) & -\frac{\cosh\beta\omega(\mathbf{k})}{\sinh\beta\omega(\mathbf{k})}\omega(\mathbf{k}) \end{pmatrix}$$

2PI Effective action

Use method “2 particle irreducible closed-time-path formalism” introduced By E.Calzetta and B.L.Hu(1988).

2PI Effective action

$$\Gamma = \frac{1}{2}\log\det\left(G_N\right)^{-1} + \frac{1}{2}\sum_{i,j=1}^{2}\log\det\left(G_{ij}\right)^{-1} + \frac{1}{2}\frac{\partial^2 S}{\partial N^a(x)\partial N^b(y)}G_N^{ab}(x,y)$$

$$+\frac{1}{2}\sum_{i,j=1}^{2}\frac{\partial^2 S}{\partial\phi_i^a(x)\partial\phi_j^b(y)}G_{ij}^{ab}(x,y) + \Gamma_2 + const$$

S is action

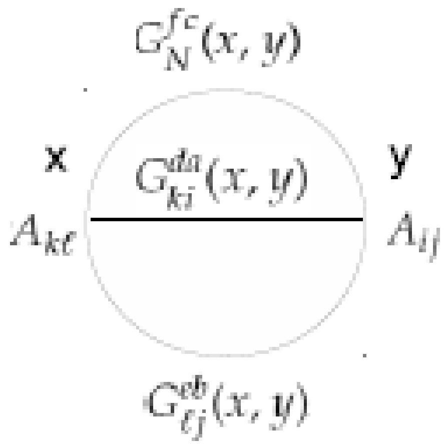
$$S[\phi^a, N^a] = \int d^4x \frac{1}{2}c_{ab}\partial_\mu N^a\partial^\mu N^b - \frac{1}{2}c_{ab}N^aN^b + \frac{1}{2}c_{ab}\partial_\mu\phi_i^a\partial^\mu\phi_j^b - \frac{1}{2}c_{ab}(m_1^2\phi_1^a\phi_1^b + m_2^2\phi_2^a\phi_2^b),$$

$$+D_{abc}\phi_i^a A_{ij}\phi_j^b N^c$$

$$c_{11} = 1,\ \ c_{22} = -1$$
$$D_{111} = 1,\ \ D_{222} = -1$$

Other component is 0

Γ_2 *describe 2PI diagram.*



Include only the diagram in left figure.
Ignore higher order diagrams.

The stationary condition of the Γ leads to the Schwinger-Dyson equation.

$$
\begin{aligned}
i\delta^{\beta\gamma}\delta_{mn}\delta(x-y) = \quad & -c_{\alpha\beta}\left(\partial_{x_\mu}^2 + m_m^2\right)G_{mn}^{\alpha\gamma}(x,y) - \int d^4z\Sigma_{m\ell}^{\beta\alpha}(x,z)G_{\ell n}^{\alpha\gamma}(z,y) \\
& + \int d^4zK_{m\ell}^{\beta\alpha}(x,z)G_{\ell n}^{\alpha\gamma}(z,y), \\
i\delta^{\delta\alpha}\delta_{mn}\delta(x-y) = \quad & -c_{\alpha\beta}\left(\partial_{y_\mu}^2 + m_n^2\right)G_{mn}^{\delta\beta}(x,y) - \int d^4zG_{m\ell}^{\delta\beta}(x,z)\Sigma_{\ell n}^{\beta\alpha}(z,y) \\
& + \int d^4zG_{m\ell}^{\delta\beta}(x,z)K_{\ell n}^{\beta\alpha}(z,y),
\end{aligned}
$$

Case without interaction terms

If there aren't interaction terms. We can derive all Green functions exactly.

$$
\begin{aligned}
G_{12}(x^0, y^0, \mathbf{k}) =& \frac{\sinh \beta\mu}{2\omega_2(\mathbf{k})\left(\cosh \beta\mu - \cosh \beta\omega\right)} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \cos[\omega_1(\mathbf{k})x^0] \sin[\omega_2(\mathbf{k})y^0] \\
&- \frac{\sinh \beta\mu}{2\omega_1(\mathbf{k})\left(\cosh \beta\mu - \cosh \beta\omega\right)} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \cos[\omega_2(\mathbf{k})y^0] \sin[\omega_1(\mathbf{k})x^0]
\end{aligned}
$$

$$
\begin{aligned}
G_{21}(x^0, y^0, \mathbf{k}) =& \frac{\sinh \beta\mu}{2\omega_1\left(\cosh \beta\omega(\mathbf{k}) - \cosh \beta\mu\right)} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \cos[\omega_2(\mathbf{k})x^0] \sin[\omega_1(\mathbf{k})y^0] \\
&- \frac{\sinh \beta\mu}{2\omega_2\left(\cosh \beta\omega(\mathbf{k}) - \cosh \beta\mu\right)} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \cos[\omega_2(\mathbf{k})y^0] \sin[\omega_1(\mathbf{k})x^0]
\end{aligned}
\quad (1)
$$

$\vdots$

We use these green functions in perturbative calculation.

Boltzmann equation

$$\partial_\mu \langle j^\mu(X)\rangle = 2B^2 G^{12}_{12}(X,X) - \int d^4z \Big(\Sigma^{1\alpha}_{1\ell}(x,z) G^{\alpha 2}_{\ell 2}(z,y) + G^{1\beta}_{1\ell}(x,z)\Sigma^{\beta 2}_{\ell 2}(z,y) \Big) + \int d^4z \Big(K^{1\alpha}_{1\ell}(x,z) G^{\alpha 2}_{\ell 2}(x,z) + G^{1\beta}_{1\ell}(x,z) K^{\beta 2}_{\ell 2}(z,y) \Big)$$

We set chemical potential μ=0 and focus on the particle number production from the interaction Hamiltonian.

$$G_{21}(x,y) = 0, G_{12}(x,y) = 0$$

Off diagonal element of green function vanish.

Solving Boltzmann equation

Boltzmann equation in perturbation.

$$\partial_\mu \langle j^\mu(X)\rangle = 2B^2 G^{12}_{12}(X,X)$$
$$-8A_{1j}A_{2j} Im \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3p}{(2\pi)^3} \int dz^0 G^{12}_{jj}(z^0, X^0, \mathbf{p}) G^{12}_N(z^0, X^0, -\mathbf{p}-\mathbf{k})$$
$$\times \Big(G^{12}_{11}(z^0, X^0, \mathbf{k}) - G^{12}_{11}(z^0, X^0, \mathbf{k}) \Big)$$

Insert Green function for free Hamiltonian to the Boltzmann equation.

Approximation

It is hard to solve this equation exactly.

We introduce approximation which is good for large time behavior.

Consider the case of the small mass difference of Φ1 and Φ2.

$$\frac{m_2^2 - m_1^2}{m_2^2 + m_1^2} = \frac{B^2}{m_\phi^2} < 1$$

We are interested in large time behavior of production rate.

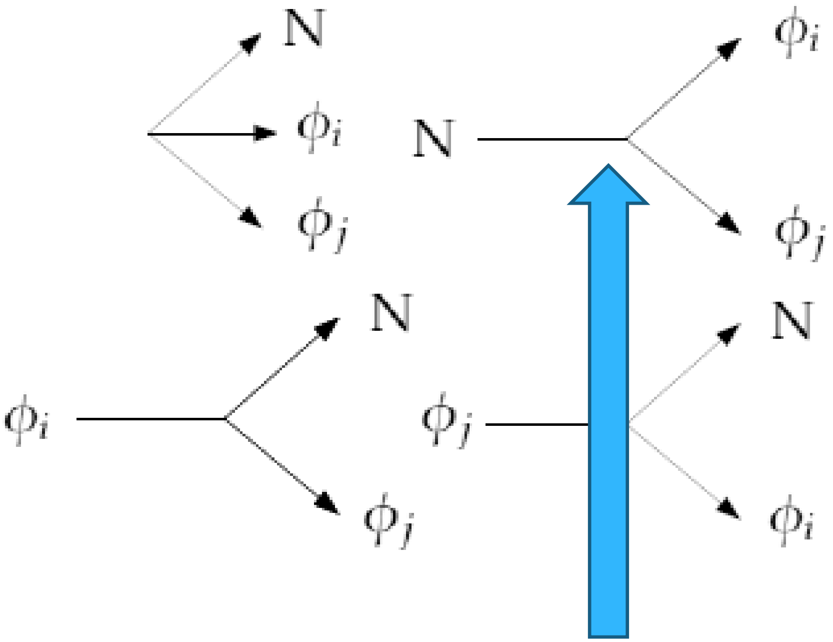
B^2 is very small.

X^0 is very large.

Define the New time t. $$t = B^2 X^0$$

Terms which proportional to B^2 can't survive without X^0 .

The terms with B, only the B^2 X^0 can survive.



Only the process can survive after large time.

After large time approximation

By using large time approximation, The Boltzmann equation become simple equation.

$$\partial_{X_\mu}\langle j^\mu(X)\rangle = 2B^2 G^{12}_{12}(X,X) - 4\pi|A|A_\phi \sin[\phi_A]$$

$$\times \int \frac{d^3p}{(2\pi^3)2\omega(\mathbf{p})2\omega_N} \int \frac{d^3p}{(2\pi^3)2\omega(\mathbf{k})} \delta(\omega_N(-\mathbf{k}-\mathbf{p}) - \omega(\mathbf{k}) - \omega(\mathbf{p}))$$

$$\times \Big(n(\mathbf{k})+1\Big) n_N \Big(n(\mathbf{p})+1\Big) \sin\left\{\left(\frac{B^2}{2\omega(\mathbf{k})} + \frac{B^2}{2\omega(\mathbf{p})}\right)t\right\}$$

We going to compute red part.

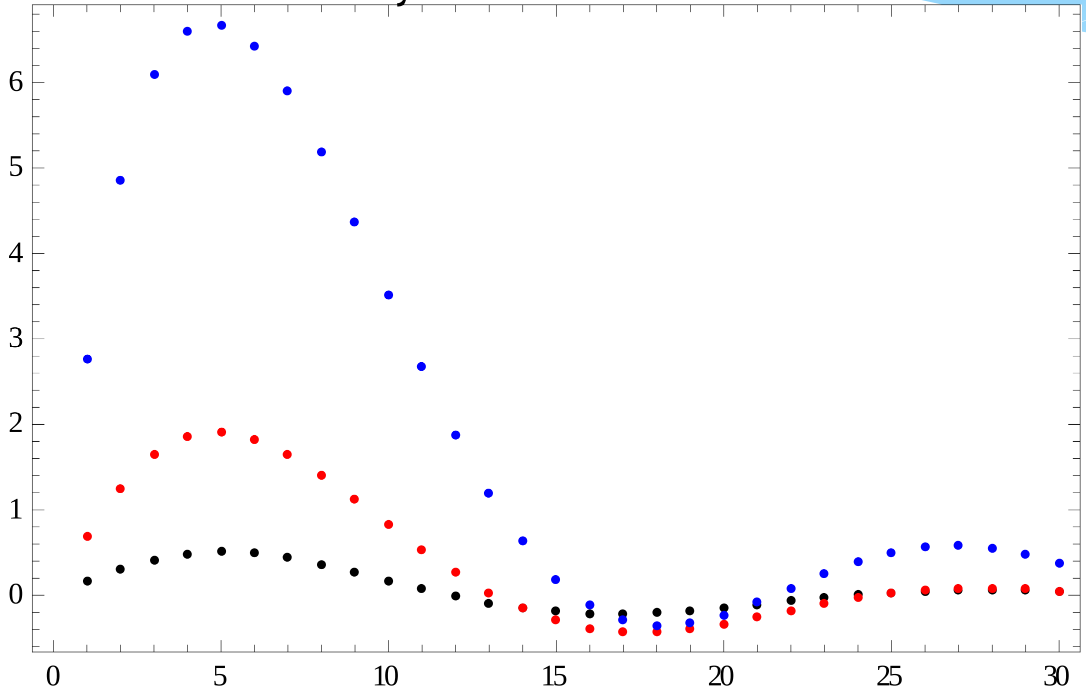
The definition of energy ω.

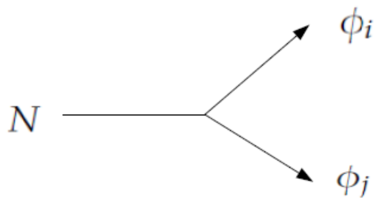
$$\omega(\mathbf{k}) = \sqrt{m_\phi^2 + \mathbf{k}^2}$$

Numerical result(from red part)

Time variation of the production rate of particle number density

BLUE>RED>BLACK
In temperature T.





$2B^2G^{12}_{12}(X,X)$?

- We haven't finished calculation of contribution from decay process in the $2B^2G^{12}_{12}(X,X)$.

However

- We know Green function's form after the approximation which is same as previous calculation .

$$2B^2G^{12}_{12}(X,X) = A_{1j}A_{2j}\frac{B^2}{\pi^2}\int dk\int dp\int_{-1}^{1} dz\frac{\omega_N}{2\omega^2(k)2\omega(p)}\delta^2(z-z_0)(z-z_0)B'_{-+}|_{z=z^0}$$

$$B_{-+} = \left\{(n_k+1)n_N(n_p+1) - n_k(n_N+1)n_p\right\}$$

We have checked production rate cancel each other between decay and inverse decay process. But we haven't understood contribution from decay process.(Future problem)

Summary

- We study the mechanism of the particle number production in the model which is CP violating .

- We show that time evolution and mechanism of the particle number production in each process.

- The sum of the contribution from decay process and inverse decay process vanishes. This is the expected result, since we consider the case for thermal equilibrium in flat-space.

- Next step, we will study the model in curved space-time (non-equilibrium). We will also study the model with fermion.